Self-Dual $N = 1$ SUSY Gauge Theories

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Abstract

We present a large set of new self-dual $N = 1$ SUSY gauge theories. Examples include $SU(N)$ theories with tensors and $SO(N)$ theories with spinors. Using these dualities as starting points, new non-trivial duals can be derived by higgsing the gauge group or by integrating out matter. General lessons that can be learned from these duals are: "accidental" infrared symmetries play an important role in duality, many theories have more than one "dual", and there seems to be no simple organizing pattern which relates duals of theories with different number of flavors.

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1 Introduction

Following the pioneering work by Seiberg on duality for supersymmetric QCD (SQCD) [1], a number of examples of duality in non-Abelian $N = 1$ supersymmetric gauge theories have been found [2-13]. The well known examples are for theories with matter in the fundamental representation of the gauge group [1-4], but there are also examples for theories with spinor representations [5].

Generalizing these dualities to theories with more general matter representations has proven to be very difficult. For theories with a tensor representation, much progress has been made by adding suitable superpotential terms which give a mass to the tensor away from the origin of moduli space [6]. Then out in moduli space the theories reduce to theories with fundamentals only, to which dual descriptions are known. Continuing these duals to the origin of moduli space, duals for the theory with the tensor and the superpotential terms were found. One might be tempted to try to obtain the theory without a superpotential by taking the coupling of the superpotential term to zero. However, it is clear that this limit is singular, since now there are components of the tensor which become massless even on generic points of moduli space. A weakly coupled low energy description which includes all massless modes still has not been found.

In this letter we propose a number of new dualities for SU theories with tensor matter and SO theories with spinors, both with no tree-level superpotential. Our proposed duals have the interesting feature that they are self-duals. The electric theory and its magnetic dual have identical gauge groups and gauge variant matter fields. In addition to the dual gauge degrees of freedom the magnetic theories also contain some number of fundamental gauge invariant "meson" fields which are coupled to the gauge variant fields in the superpotential.

Self-duals have been known to exist for theories with matter only in fundamental representations [1, 2, 3, 7, 8]. For example, consider SQCD with N colors and the special number of flavors $F = 2N$. The field content and symmetry properties of this theory are summarized in the following table.

$$
\begin{array}{c|c|ccccc}
 & SU(N) & SU(2N) & SU(2N) & U(1) & U(1)_R \\
\hline
Q & \square & \square & 1 & 1 & 1/2 \\
\bar{Q} & \bar{\square} & 1 & \square & -1 & 1/2 \\
\end{array}
\tag{1}
$$

The dual for this theory is just a special case of Seiberg's dual for SQCD. It is also an $SU(N)$ gauge theory with dual quarks q and $\overline{q}$ and a fundamental "meson" gauge singlet field M coupled in the superpotential with the term $W = Mq\bar{q}$.

	$SU(N)$	$SU(2N)$	$SU(2N)$	$U(1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	1	1	$1/2$
$\bar{q}$	$\overline{\square}$	1	$\overline{\square}$	-1	$1/2$
M	1	$\square$	$\square$	0	1

$$(2)$$

Note that the standard consistency checks are rather trivial for this self-dual. For example, the anomaly matching conditions are almost all trivially satisfied. All anomalies involving $U(1)$ and $U(1)_R$ charges are matched because the fermion component of M is uncharged under the $U(1)$'s and the contributions of the dual quarks are identical to the contributions of the electric quarks. The only non-trivially matched anomalies are the $SU(2N)^3$ non-Abelian flavor anomalies. The operator maps of the gauge invariant "mesons" and "baryons" are also very simple:

$$
\begin{aligned}
Q\bar{Q} &\leftrightarrow M \\
Q^N &\leftrightarrow q^N \\
\bar{Q}^N &\leftrightarrow \bar{q}^N
\end{aligned}
$$

$$(3)$$

The dual theory has an additional gauge invariant $q\bar{q}$ which is set to zero by the equations of motion derived from the superpotential. Since these operators parameterize the moduli spaces of the two theories, this operator map is necessary to show the equivalence of the moduli spaces. Note that one can obtain the dualities for all other numbers of flavors by integrating out flavors from one of the two theories.

Similar self-duals exist for theories with "quarks" transforming as fundamentals of Sp [3], SO [2], and some exceptional gauge groups [7, 8]. Theories with matter in tensor representations and certain simplifying tree-level superpotentials have also been found to have self-duals [6]. More recently, a self-dual has been proposed for a theory with a tensor field and no tree-level superpotential [9]. Here, we present similar duals for a number of theories with matter in tensor and spinor representations.

In the following section we review Seiberg's proposed dual of $SU(2)$ SQCD, which requires the existence of accidental infrared symmetries. We

then describe three different duals of this theory. In the third section we list a number of new self-duals and other duals which can be derived via flows from the self-duals. For some of the presented dualities we also give detailed consistency checks. While the number of self-duals that we present is quite large, it is clear that this list is not exhaustive and it should be possible to generate more examples with similar properties. We present our conclusions in the final section.

2 Multiple self-duals and accidental symmetries

In this section we review two apparently common but not very well known features of duality. One is the occurrence of accidental infrared symmetries [8, 10], and the other is the existence of multiple "duals" for a single theory [12, 13].

Frequently, the ultraviolet description of one (or several) of these duals does not manifestly have the full global symmetry of the infrared. Only in the far infrared, below the intrinsic scale of the strong gauge interactions, does the full global symmetry become manifest. Let us illustrate this with the example of $SU(2)$ SQCD for which the fundamental and antifundamental representations are equivalent. Thus, the full non-Abelian flavor symmetry is $SU(2F)$ rather than just $SU(F) \times SU(F) \times U(1)$, and the spectrum of the theory falls into $SU(2F)$ representations for all numbers of flavors F. The independent gauge invariant chiral operators which parameterize the moduli space and correspond to infrared degrees of freedom are contained in the matrix $A = QQ$ which transforms in the antisymmetric two index tensor representation of $SU(2F)$.

In Seiberg's dual description, discussed in the previous section, the full flavor symmetry is not manifest, and one might be worried that the duality is not valid for $SU(2)$. Seiberg's proposed dual and its ultraviolet-symmetry for arbitrary F are

$$
\begin{array}{c|c|cccc}
 & SU(F-2) & SU(F) & SU(F) & U(1) & U(1)_R \\
\hline
q & \square & \overline{\square} & 1 & 2/(F-2) & 2/F \\
\bar{q} & \overline{\square} & 1 & \overline{\square} & -2/(F-2) & 2/F \\
M & 1 & \square & \square & 0 & 2-4/F
\end{array}
\tag{4}
$$

3

Thus the ultraviolet global symmetries of this description contain only $SU(F) \times SU(F) \times U(1) \subset SU(2F)$. In order for the infrared spectrum to respect the full $SU(2F)$ flavor symmetry the "meson" M has to be complemented by bound states of the dynamical dual quarks q and $\bar{q}$. Together these degrees of freedom transform as a full $SU(2F)$ representation. Thus, the $SU(2F)$ symmetry generators mix fundamental fields with composites. In order for this to make any sense, the fields which are to form a complete irreducible $SU(2F)$ representation have to have identical quantum numbers under the other symmetries. Here, these extra quantum numbers are the $U(1)_R$ charges. The chiral gauge invariants of the dual with their global transformation properties are

$$
\begin{array}{c|cccc}
 & SU(F) & SU(F) & U(1) & U(1)_R \\
\hline
M & \square & \square & 0 & 2-4/F \\
b = q^{F-2} & {\begin{smallmatrix}\square\\\square\end{smallmatrix}} & 1 & 2 & 2-4/F \\
\bar{b} = \bar{q}^{F-2} & 1 & {\begin{smallmatrix}\square\\\square\end{smallmatrix}} & -2 & 2-4/F
\end{array}
\tag{5}
$$

The composite operator $q\bar{q}$ is set to zero by the M equation of motion. Note that the $U(1)_R$ charges of the all the non-vanishing chiral operators are identical, thus it is possible to unify M, b, and $\bar{b}$ into the antisymmetric tensor representation of $SU(2F)$.

Another necessary condition for the emergence of the full $SU(2F)$ global symmetry is that the scaling dimensions of the three operators M, b and $\bar{b}$ have to agree. In the range of F where this theory has an infrared fixed-point, the theory is superconformal, and the scaling dimensions are given by $3/2$ times the superconformal R-charges of the fields. Since the R-charges commute with $SU(2F)$, the dimensions respect the full flavor symmetry as well. Note that the $SU(2)$ theory and its dual can be reached via flows from the more general $SU(N)$ theory and its $SU(F-N)$ dual, thus all the usual consistency checks also apply to this dual.

There is an additional dual of the $SU(2)$ theory which has a manifest $SU(2F)$ symmetry. The field content of this Sp dual is

$$
\begin{array}{c|c|cc}
 & Sp(2F-6) & SU(2F) & U(1)_R \\
\hline
q & \square & \bar{\square} & 2/F \\
A & 1 & {\begin{smallmatrix}\square\\\square\end{smallmatrix}} & 2-4/F
\end{array}
\tag{6}
$$

In this dual the whole antisymmetric tensor $A = QQ$ is fundamental.

An interesting special case is the theory with $F = 4$. This theory is self-dual, and both the SU-dual and the Sp-dual have an $SU(2)$ gauge group. The two duals differ in the ultraviolet by their "meson" content. While the Sp dual has a "meson" $A = QQ$ which transforms as an antisymmetric tensor of the full $SU(8)$ flavor symmetry, the SU dual only has an $SU(4) \times SU(4)$ symmetry and the "meson" transforms as $M = (\square, \square)$. Again, in the infrared, the full flavor symmetry is restored and the "meson" M is unified into an $SU(8)$ multiplet together with the composites $b = qq$ and $\bar{b} = \bar{q}\bar{q}$. There is a third possible dual which also has an $SU(2)$ gauge symmetry, and where the only fundamental "meson" fields are the "baryons" $B = (\substack{\square\\\square}, 1)$ and $\bar{B} = (1, \substack{\square\\\square})$.

We summarize the field content of the self-dual theory with its three duals in the following table.

	$SU(2)$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	1	1	$1/2$
$\bar{Q}$	$\square$	1	$\square$	-1	$1/2$
	$SU(2)_{D1}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
q	$\square$	$\bar{\square}$	1	1	$1/2$
$\bar{q}$	$\square$	1	$\bar{\square}$	-1	$1/2$
M	1	$\square$	$\square$	0	1
	$SU(2)_{D2}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
q	$\square$	$\bar{\square}$	1	-1	$1/2$
$\bar{q}$	$\square$	1	$\bar{\square}$	1	$1/2$
M	1	$\square$	$\square$	0	1
B	1	$\substack{\square\\\square}$	1	2	1
$\bar{B}$	1	1	$\substack{\square\\\square}$	-2	1
	$SU(2)_{D3}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
q	$\square$	$\square$	1	-1	$1/2$
$\bar{q}$	$\square$	1	$\square$	1	$1/2$
B	1	$\substack{\square\\\square}$	1	2	1
$\bar{B}$	1	1	$\substack{\square\\\square}$	-2	1

$$(7)$$

The original theory and dual number 2 in this table have a manifest $SU(2F)$ flavor symmetry. For purpose of comparison between the three duals we listed only the transformation properties under the $SU(F) \times SU(F) \times U(1)$ subgroup. Note that the charges and representations of the dual quarks

q and $\bar{q}$ are different in the three duals. Also, these duals have differing superpotentials

$$
\begin{aligned}
W_{D_1} &= Mq\bar{q} \\
W_{D_2} &= Mq\bar{q} + Bq^2 + \bar{B}\bar{q}^2 \\
W_{D_3} &= Bq^2 + \bar{B}\bar{q}^2 \ .
\end{aligned}
\tag{8}
$$

As we will demonstrate in the following section, these phenomena are quite common for self-dual theories. We will present several new examples of self-dual theories with more than one dual, and duals which only have a subset of the full flavor symmetry manifest in the ultraviolet.

3   New examples of self-duals

3.1 $SU(8)$ with two antisymmetric tensors and 8 anti-fundamentals

The first new example of self-dual theories that we present is based on an $SU(8)$ gauge group with matter content $2\,\square\hspace{-0.2em}\square + 8\,\bar{\square}$. The field content and the global symmetries of this theory and its dual is summarized in the table below.

	$SU(8)$	$SU(2)$	$SU(8)$	$U(1)$	$U(1)_R$
A	$\square\hspace{-0.2em}\square$	$\square$	1	2	0
$\bar{Q}$	$\bar{\square}$	1	$\square$	-3	$\frac{1}{2}$
	$SU(8)$	$SU(2)$	$SU(8)$	$U(1)$	$U(1)_R$
a	$\square\hspace{-0.2em}\square$	$\square$	1	2	0
$\bar{q}$	$\bar{\square}$	1	$\bar{\square}$	-3	$\frac{1}{2}$
$H = (A\bar{Q}\bar{Q})$	1	$\square$	$\square\hspace{-0.2em}\square$	-4	1
$K = (A^5\bar{Q}\bar{Q})$	1	$\square$	$\square\hspace{-0.2em}\square$	4	1

(9)

The tree-level superpotential in the dual theory is [3]

$$
W = Ha^5\bar{q}\bar{q} + Ka\bar{q}\bar{q}.
\tag{10}
$$

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[3] In this paper we absorb the numerical coefficients of the different superpotential terms into field redefinitions.

The 't Hooft anomaly matching conditions are almost trivially satisfied, since the extra gauge singlets do not contribute to any anomaly involving $U(1)_R$ while the $U(1)$ charges of the gauge singlets come in charge conjugate pairs. Thus the only non-trivial anomaly that is left to check is $SU(8)^3$ which matches between the electric and magnetic descriptions.

The correspondence of flat directions of the electric and magnetic theories is straightforward as well:

$$A^4 \leftrightarrow a^4$$
$$\bar{Q}^8 \leftrightarrow \bar{q}^8$$
$$A\bar{Q}^2 \leftrightarrow H$$
$$A^5\bar{Q}^2 \leftrightarrow K$$

The additional gauge invariants of the magnetic theory, $a\bar{q}^2$ and $a^5\bar{q}^2$ are set to zero by the H and K equations of motion.

Further consistency checks of this proposed duality include higgsing the gauge group by going out in different directions on the moduli space[4]. First we consider giving a vacuum expectation value (VEV) to one of the antisymmetric tensor fields

$$A_1 = v \begin{pmatrix} i\sigma_2 & & & \\ & i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & i\sigma_2 \end{pmatrix}. \tag{11}$$

This breaks the gauge group to $Sp(8)$, and the uneaten fields transforming under $Sp(8)$ are $\boxminus + 8\,\square$, with no superpotential. The operator map determines that in the dual this corresponds to giving a VEV to

$$a_1 = w \begin{pmatrix} i\sigma_2 & & & \\ & i\sigma_2 & & \\ & & i\sigma_2 & \\ & & & i\sigma_2 \end{pmatrix}, \tag{12}$$

which breaks the gauge group to $Sp(8)$ as well. The particle content of the

[4] Since this $SU(8)$ theory is completely chiral, one can not add mass terms for any of the operators.

magnetic description is

$$
\begin{array}{c|c|c}
 & Sp(8) & SU(8) \\
\hline
a & \square\!\square & 1 \\
q & \square & \overline{\square} \\
M_0 \equiv H_1 & 1 & \\
M_1 \equiv H_2 & 1 & \\
M_2 \equiv K_1 & 1 & \\
M_3 \equiv K_2 & 1 & \\
\end{array}
\qquad,
\tag{13}
$$

where we have decomposed H and K under the explicitly broken $SU(2)$ global symmetry. The superpotential of equation 10 becomes

$$
W = M_0 q a^3 q + M_1 q a^2 q + M_2 q a q + M_3 q q
\tag{14}
$$

after substituting the VEV of a_1 from equation 12. Thus we recover the self-dual of $Sp(8)$ with $\square\!\square + 8\,\square$ of Ref. [9].

Another possible way to higgs the electric theory is by giving a VEV to $A_1 \bar{Q}_7 \bar{Q}_8$. This breaks the gauge group to $SU(6)$ and the remaining matter content transforming under $SU(6)$ is $2\,\square\!\square + 2\,\square + 6\,\overline{\square}$. On the magnetic side this corresponds to giving a VEV to the $(7,8)$ component of H_1, which explicitly breaks some of the global symmetries, but not the $SU(8)$ gauge symmetry. Thus, we obtain a non-trivial dual of $SU(6)$ with $2\,\square\!\square + 2\,\square + 6\,\overline{\square}$, where the dual description is in terms of an $SU(8)$ gauge group with $2\,\square\!\square + 8\,\overline{\square} +$ "mesons", and a complicated tree-level superpotential. Similar non-trivial dualities derived from self-duals will be described in detail in the next section.

3.2 $SU(6)$ with a three index antisymmetric tensor

In this section we present a self-dual description for an $SU(6)$ gauge theory with a three-index tensor and six flavors, $\square\!\square\!\square + 6(\square + \overline{\square})$. Starting from this duality we derive several other non-trivial dualities by either higgsing the electric gauge group or integrating out flavors. We will present these dualities in detail. An important lesson to be learned from this set of dualities is that in general dual theories do not follow any simple pattern. As we will see, the tensor representations of the electric and magnetic descriptions can be very different. Furthermore, theories with identical gauge group and gauge degrees

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of freedom but with slightly differing "meson" content and superpotential are dual to very different electric theories.

The field content and symmetries of the electric and magnetic theories are

	$SU(6)$	$SU(6)_Q$	$SU(6)_{\bar Q}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	1	0	-2	0
Q	$\square$	$\square$	1	1	1	$\frac{1}{2}$
$\bar Q$	$\bar\square$	1	$\square$	-1	1	$\frac{1}{2}$

	$SU(6)_D$	$SU(6)_Q$	$SU(6)_{\bar Q}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	1	0	-2	0
q	$\square$	$\bar\square$	1	1	1	$\frac{1}{2}$
$\bar q$	$\bar\square$	1	$\bar\square$	-1	1	$\frac{1}{2}$
M_0	1	$\square$	$\square$	0	2	1
M_2	1	$\square$	$\square$	0	-2	1

$$\tag{15}$$

The dual theory has the following tree-level superpotential

$$W = M_0 q a^2 \bar q + M_2 q \bar q. \tag{16}$$

The global anomalies in the two descriptions again match almost trivially. Flat directions in the two theories are also in one-to-one correspondence. The mapping of the flat directions is given in the list below

$$
\begin{array}{ll}
AQ^3 \leftrightarrow aq^3 & Q^6 \leftrightarrow q^6 \\
A\bar Q^3 \leftrightarrow a\bar q^3 & \bar Q^6 \leftrightarrow \bar q^6 \\
A^3 Q^3 \leftrightarrow a^3 q^3 & Q\bar Q \leftrightarrow M_0 \\
A^3 \bar Q^3 \leftrightarrow a^3 \bar q^3 & Q A^2 \bar Q \leftrightarrow M_2 \\
A^4 \leftrightarrow a^4 &
\end{array}
\tag{17}
$$

The extra $qa^2\bar q$ and $q\bar q$ directions in the magnetic theory are lifted by the M_0 and M_2 equations of motion.

Starting from the above presented dual for $SU(6)$ with $\begin{smallmatrix}\square\\\square\end{smallmatrix} + 6(\square + \bar\square)$ one can obtain new dualities by integrating out a flavor or higgsing $SU(6)$ to $SU(5)$ or $SU(4)$. First, we consider integrating out one flavor. The electric theory will be $SU(6)$ with $\begin{smallmatrix}\square\\\square\end{smallmatrix} + 5(\square + \bar\square)$. In the dual description the quark mass term maps onto the superpotential term $m(M_0)_{66}$. Thus

$$W = M_0 q a^2 \bar q + M_2 q \bar q + m(M_0)_{66}. \tag{18}$$

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The M_0 and M_2 equations of motion force a VEV for the fields $(q_6)_6 = (\bar{q}_6)_6 = a_{123} = a_{145} = v$ and zero VEVs for all other components. This breaks the $SU(6)$ gauge group to $Sp(4)$. The resulting pair of dual theories is given in the table below.

	$SU(6)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\boxminus$	1	1	0	-5	-1
Q	$\square$	$\square$	1	1	3	1
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	3	1
	$Sp(4)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\boxminus$	1	1	0	-10	-2
q	$\square$	$\bar{\square}$	1	$3/2$	2	1
$\bar{q}$	$\bar{\square}$	1	$\bar{\square}$	$-3/2$	2	1
M_0	1	$\square$	$\square$	0	6	2
M_2	1	$\square$	$\square$	0	-4	0

$$(19)$$

The new superpotential for the magnetic $Sp(4)$ theory is

$$W = M_0 q a \bar{q} + M_2 q \bar{q}. \tag{20}$$

As a consistency check on the presented dual description we integrate out one more flavor in the $SU(6)$ theory with a $\boxminus + 5(\square + \bar{\square})$. This theory with four flavors is known to be confining [11], and we should reproduce its spectrum and the confining superpotential from the dual description. The $SU(6)$ theory with four flavors has the following confined infrared spectrum: $M_0 = Q\bar{Q}$, $M_2 = QA^2\bar{Q}$, $B_1 = AQ^3$, $\bar{B}_1 = A\bar{Q}^3$, $B_3 = A^3Q^3$, $\bar{B}_3 = A^3\bar{Q}^3$ and $T = A^4$. The confining superpotential in terms of these fields is given by

$$
\begin{aligned}
W_{F=4} &= \frac{1}{\Lambda^{11}}\Big(M_0 B_1 \bar{B}_1 T + B_3 \bar{B}_3 M_0 + M_2^3 M_0 + T M_2 M_0^3 + \\
&\quad \bar{B}_1 B_3 M_2 + B_1 \bar{B}_3 M_2 \Big),
\end{aligned}
\tag{21}
$$

In the dual $Sp(4)$ description, the mass term forces non zero VEVs for a and q_5, $\bar{q}_5$, which completely breaks the $Sp(4)$ gauge group. The fields $(M_0)_{55}, (M_0)_{5i}, (M_0)_{i5}, (M_2)_{55}, (M_0)_{5i}, (M_2)_{i5}$ and two components of q_i, $\bar{q}_i$ as well as two uneaten singlet components of a get masses from the tree-level superpotential. The remaining components of q_i, $\bar{q}_i$ are identified with B_1, $\bar{B}_1$, B_3 and $\bar{B}_3$, the remaining singlet with T, and the remaining components

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of M_0 and M_2 with the corresponding M_0 and M_2 in the $SU(6)$ theory. The tree-level superpotential reproduces the terms $M_0 B_1 \bar{B}_1 T$, $B_3 \bar{B}_3 M_0$, $\bar{B}_1 B_3 M_2$ and $B_1 \bar{B}_3 M_2$ of the confining superpotential of equation 21, while the missing $M_2^3 M_0$ and $T M_2 M_0^3$ terms are presumably generated by instanton effects in the completely broken $Sp(4)$.

Using the dual pair of the table in Eq. 19 one can obtain a dual description for $Sp(4)$ with an antisymmetric tensor and ten fundamentals. We add conjugate "meson" fields $\overline{M}_0$ and $\overline{M}_2$ with mass terms $W = M_0 \overline{M}_0 + M_2 \overline{M}_2$ to both theories. Integrating out these massive "mesons" in the $Sp(4)$ theory will set the superpotential to zero. In the $SU(6)$ theory these mass terms correspond to non-trivial interaction terms. The resulting dual of the $Sp(4)$ theory with an antisymmetric tensor and ten fundamentals is given in the following table.

	$Sp(4)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\boxminus$	1	1	0	-10	-2
q	$\square$	$\square$	1	$3/2$	2	1
$\bar{q}$	$\overline{\square}$	1	$\square$	$-3/2$	2	1

	$SU(6)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\boxminus$	1	1	0	-5	-1
Q	$\square$	$\overline{\square}$	1	1	3	1
$\bar{Q}$	$\overline{\square}$	1	$\overline{\square}$	-1	3	1
$\overline{M}_0$	1	$\square$	$\square$	0	-6	0
$\overline{M}_2$	1	$\square$	$\square$	0	4	2

$$\text{(22)}$$

The dual has the tree-level superpotential

$$W = \overline{M}_0 Q \bar{Q} + \overline{M}_2 Q A^2 \bar{Q} \qquad (23)$$

in the magnetic $SU(6)$ theory. Note that the electric $Sp(4)$ has an $SU(10)$ flavor symmetry, while naïvely the magnetic $SU(6)$ theory has only $SU(5) \times SU(5) \times U(1)$. This is another example of accidental symmetries analogous to the $SU(F-2)$ dual of $SU(2)$ described in detail in Section 2.

Next, we obtain dual descriptions of $SU(5)$ with matter content $\boxminus + \overline{\boxminus} + F(\square + \overline{\square})$ with $F = 4$ or 5 by higgsing the $SU(6)$ theory with a three-index tensor and six flavors, which was described above. We give a VEV to one flavor breaking $SU(6)$ to $SU(5)$. The three-index antisymmetric tensor decomposes into $\boxminus$ and $\overline{\boxminus}$ and there are five $SU(5)$ flavors that remain uneaten.

There remain also $SU(5)$ singlets which can be eliminated by adding conjugate singlets and terms

$$W = S_i(q_i\bar{q}_6) + \bar{S}_i(q_6\bar{q}_i), \quad i = 1,\ldots,5, \qquad (24)$$

to the superpotential. This extra term makes the unwanted fields massive after q_6 and $\bar{q}_6$ get a VEV. In the dual description these terms correspond to mass terms $S_i(M_0)_{i6}$ and $\bar{S}_i(M_0)_{6i}$, which after integrating out these fields set all superpotential terms involving $(M_0)_{i6}$ and $(M_0)_{6i}$ to zero. The VEV of $(M_0)_{66}$ explicitly breaks the $SU(6) \times SU(6)$ global symmetry to $SU(5) \times SU(5)$. There are three non-anomalous $U(1)$ symmetries preserved by the tree-level superpotential and the VEV of M_0. Thus, the resulting dual pair is

	$SU(5)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
A	$\Box\!\Box$	1	1	0	-5	$-\frac{3}{5}$	0
$\bar{A}$	$\overline{\Box\!\Box}$	1	1	0	-5	$\frac{3}{5}$	0
Q	$\Box$	$\Box$	1	1	3	$\frac{1}{5}$	$\frac{3}{5}$
$\bar{Q}$	$\overline{\Box}$	1	$\Box$	-1	3	$-\frac{1}{5}$	$\frac{3}{5}$

	$SU(6)$	$SU(5)_Q$	$SU(5)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
a	$\Box\!\Box$	1	1	0	-5	0	0
q	$\Box$	$\overline{\Box}$	1	$\frac{2}{3}$	2	$\frac{1}{3}$	$\frac{2}{5}$
$\bar{q}$	$\overline{\Box}$	1	$\overline{\Box}$	$-\frac{2}{3}$	2	$-\frac{1}{3}$	$\frac{2}{5}$
q_6	$\Box$	1	1	$\frac{5}{3}$	5	$-\frac{2}{3}$	1
$\bar{q}_6$	$\overline{\Box}$	1	1	$-\frac{5}{3}$	5	$\frac{2}{3}$	1
M_0	1	$\Box$	$\Box$	0	6	0	$\frac{6}{5}$
M_2	1	$\Box$	$\Box$	0	-4	0	$\frac{6}{5}$
$(M_2)_{6i}$	1	1	$\Box$	-1	-7	1	$\frac{3}{5}$
$(M_2)_{i6}$	1	$\Box$	1	1	-7	-1	$\frac{3}{5}$
$(M_2)_{66}$	1	1	1	0	-10	0	0

$$(25)$$

with a superpotential in the magnetic $SU(6)$ theory

$$W = M_0 q a^2 \bar{q} + M_2 q\bar{q} + (M_2)_{6i} q_6 \bar{q}_i + (M_2)_{i6} q_i \bar{q}_6 + (M_2)_{66} q_6 \bar{q}_6 + q_6 a^2 \bar{q}_6. \quad (26)$$

Next, we construct a dual description for $SU(5)$ with $F = 4$. This dual can be obtained in two different ways: either by higgsing the $SU(6)$ theory with $\Box\!\Box + 5(\Box + \overline{\Box})$ or by integrating out a flavor from $SU(5)$ with $F = 5$. Here,

we consider the latter possibility. Adding a mass term $mq_5\bar{q}_5$ corresponds to adding a term $m(M_0)_{55}$ to the superpotential of equation 26. This gives VEVs to q_5, $\bar{q}_5$, and a in analogy to the case of integrating out a flavor from $SU(6)$ with $\boxminus+6(\square+\bar{\square})$. These VEVs break $SU(6)$ to $Sp(4)$ and the resulting dual pair is given in the table below.

	$SU(5)$	$SU(4)_Q$	$SU(4)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
A	$\boxminus$	1	1	1	0	-4	0
$\bar{A}$	$\boxminus$	1	1	-1	0	-4	0
Q	$\square$	$\square$	1	0	1	3	$\frac{1}{2}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	0	-1	3	$\frac{1}{2}$

	$Sp(4)$	$SU(4)_Q$	$SU(4)_{\bar{Q}}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
a	$\boxminus$	1	1	0	0	-8	0
q	$\square$	$\bar{\square}$	1	$-\frac{1}{2}$	1	1	$\frac{1}{2}$
$\bar{q}$	$\bar{\square}$	1	$\bar{\square}$	$\frac{1}{2}$	-1	1	$\frac{1}{2}$
q_6	$\square$	1	1	$\frac{3}{2}$	2	4	1
$\bar{q}_6$	$\bar{\square}$	1	1	$-\frac{3}{2}$	-2	4	1
M_0	1	$\square$	$\square$	0	0	6	1
M_2	1	$\square$	$\square$	0	0	-2	1
$(M_2)_{6i}$	1	1	$\square$	-2	-1	-5	$\frac{1}{2}$
$(M_2)_{i6}$	1	$\square$	1	2	1	-5	$\frac{1}{2}$
$(M_2)_{66}$	1	1	1	0	0	-8	0

$$(27)$$

The superpotential for the magnetic $Sp(4)$ theory is

$$W = M_0 q a \bar{q} + M_2 q \bar{q} + (M_2)_{6i} q_6 \bar{q}_i + (M_2)_{i6} q_i \bar{q}_6 + (M_2)_{66} q_6 \bar{q}_6 + q_6 a \bar{q}_6. \quad (28)$$

Integrating out one more flavor in the electric $SU(5)$ theory with $F = 4$ completely breaks the gauge group of the dual theory. This is in complete analogy to integrating out one flavor from the $SU(6)$ theory with $\boxminus+5(\square+\bar{\square})$: one obtains the correct confining spectrum and several terms in the confining superpotential.

There are several other non-trivial dualities which can be derived from these duals by either higgsing the gauge group or adding mass terms for quark antiquark pairs. Some of these duals are summarized in Fig. 1. The figure elucidates one of the main lesson of this section: theories with the same gauge group and gauge degrees of freedom can have very different duals,
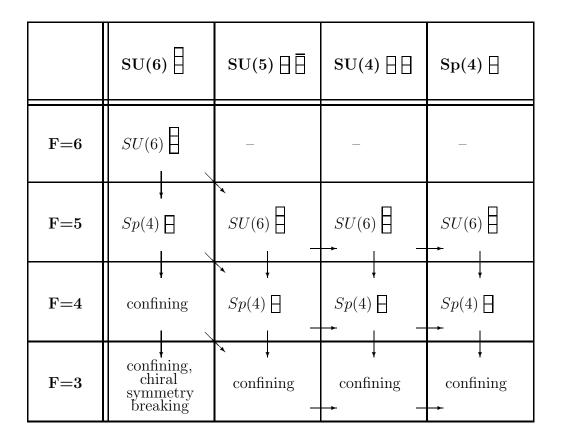
	SU(6) ▦	**SU(5)** ▦ ▦̄	**SU(4)** ▦ ▦	**Sp(4)** ▯
F=6	$SU(6)$ ▦	–	–	–
F=5	$Sp(4)$ ▯	$SU(6)$ ▯	$SU(6)$ ▯	$SU(6)$ ▯
F=4	confining	$Sp(4)$ ▦	$Sp(4)$ ▦	$Sp(4)$ ▦
F=3	confining, chiral symmetry breaking	confining	confining	confining

Figure 1: The chain of theories obtained from the self-dual of an $SU(6)$ theory with the three-index antisymmetric tensor and six flavors. The gauge group and the tensor field content of the electric theories are indicated in the top row. The first column gives the number of flavors. The confining theories are identified as such in the table, for the others we give the dual gauge group and tensor field content. The arrows depict possible flows between these theories either by higgsing the gauge group or by adding mass terms.

depending on the "meson" content and the tree-level superpotential. For example, $SU(6)$ with ▦ $+ 6(\square + \bar{\square})$ is self-dual, it is also dual to $SU(5)$ with ▦ $+ \bar{▦} + 5(\square + \bar{\square})$ and dual to $SU(4)$ with 2▦ $+ 5(\square + \bar{\square})$, depending on the "meson" content.

We see in these examples that magnetic theories with very similar "meson" fields can be dual to electric theories with radically different gauge groups and/or matter content. This suggests that finding the field content of a dual for a theory with tensors is quite difficult in general. The potential presence of accidental symmetries complicates finding duals even more, since one of the main tools for identifying dualities is to require that global

symmetries and their anomalies match in the electric and magnetic theories. When two theories have differing ultraviolet symmetries, it is difficult to decide whether there is a duality connecting the two theories, with some symmetries being accidental, or whether there is no duality at all.

3.3 Multiple self-duals

In this section we present another example of multiple dualities, similar to the $SU(2)$ theory presented in Section 2. We consider $SU(4)$ with $2\,\square\!\square + 4(\square + \bar{\square})$. An $Sp(4)$ dual for this theory can be obtained by higgsing and integrating out a flavor from the self-dual of $SU(6)$ with $\square\!\square$. This $SU(4)$ theory has however several self-dual descriptions as well. These are described in Table 1.

Note that the representations and charges of the dual quarks differ in the three duals. The 't Hooft anomaly matching and the operator map is again straightforward if one includes the appropriate superpotentials for the different duals:

$$
\begin{aligned}
W_{D_1} &= M_0 q a^2 \bar{q} + M_2 q \bar{q} + B a q^2 + \bar{B} a \bar{q}^2 \\
W_{D_2} &= M_0 q a^2 \bar{q} + M_2 q \bar{q} \\
W_{D_3} &= B a q^2 + \bar{B} a \bar{q}^2
\end{aligned}
\tag{29}
$$

As a consistency check on this duality we consider giving giving a VEV to one of the antisymmetric tensors. This breaks both the electric and the magnetic $SU(4)$ gauge group to $Sp(4)$, leaving one antisymmetric tensor and eight flavors of $Sp(4)$. A self-dual of this $Sp(4)$ theory has been described in Ref. [9]. Our D_1 self-dual exactly reproduces the dual of [9]. Our other two duals lead to new dualities for the $Sp(4)$ theory. Note that in the latter case the fundamental fields of the dual $Sp(4)$ theory do not combine into complete representations of the global $SU(8)$ group, only an $SU(4) \times SU(4) \times U(1)$ subgroup is manifest in the dual. The full $SU(8)$ global symmetry arises as an accidental symmetry of the infrared.

As a consistency check, one can further higgs the $Sp(4)$ gauge group to $SU(2) \times SU(2)$, by giving a VEV to the to the remaining antisymmetric tensor. One finds that the three duals flow to two copies of the three self-duals of $SU(2)$ with 8 doublets described in Section 2, providing another consistency check on these dualities.

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	$SU(4)$	$SU(2)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\yng(1,1)$	$\square$	1	1	0	2	0
Q	$\square$	1	$\square$	1	1	-1	$\frac{1}{2}$
$\bar{Q}$	$\overline{\square}$	1	1	$\square$	-1	-1	$\frac{1}{2}$

	$SU(4)_{D_1}$	$SU(2)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\yng(1,1)$	$\square$	1	1	0	2	0
q	$\square$	1	$\overline{\square}$	1	-1	-1	$\frac{1}{2}$
$\bar{q}$	$\overline{\square}$	1	1	$\overline{\square}$	1	$--1$	$\frac{1}{2}$
M_0	1	1	$\square$	$\square$	0	-2	1
M_2	1	1	$\square$	$\square$	0	2	1
B	1	$\square$	$\yng(1,1)$	1	2	0	1
$\bar{B}$	1	$\square$	1	$\yng(1,1)$	-2	0	1

	$SU(4)_{D_2}$	$SU(2)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\yng(1,1)$	$\square$	1	1	0	2	0
q	$\square$	1	$\overline{\square}$	1	1	-1	$\frac{1}{2}$
$\bar{q}$	$\overline{\square}$	1	1	$\overline{\square}$	-1	$--1$	$\frac{1}{2}$
M_0	1	1	$\square$	$\square$	0	-2	1
M_2	1	1	$\square$	$\square$	0	2	1

	$SU(4)_{D_3}$	$SU(2)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
a	$\yng(1,1)$	$\square$	1	1	0	2	0
q	$\square$	1	$\square$	1	-1	-1	$\frac{1}{2}$
$\bar{q}$	$\overline{\square}$	1	1	$\square$	1	$--1$	$\frac{1}{2}$
B	1	$\square$	$\yng(1,1)$	1	2	0	1
$\bar{B}$	1	$\square$	1	$\yng(1,1)$	-2	0	1

Table 1: The multiple self-duals of $SU(4)$ with $2\,\yng(1,1) + 4(\square + \overline{\square})$.

3.4 $SU(2N)$ with $\Yasymm + \overline{\Yasymm} + 4(\square + \bar\square)$

In this section we describe the generalization of one of the $SU(4)$ self-duals presented in the previous section to $SU(2N)$ with $\Yasymm + \overline{\Yasymm} + 4(\square + \bar\square)$. The pair of dual theories is given in the table below.

	$SU(2N)$	$SU(4)_Q$	$SU(4)_{\bar Q}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
A	$\Yasymm$	1	1	1	0	-4	0
$\bar A$	$\overline{\Yasymm}$	1	1	-1	0	-4	0
Q	$\square$	$\square$	1	0	1	$2N-2$	$\frac12$
$\bar Q$	$\bar\square$	1	$\square$	0	-1	$2N-2$	$\frac12$

	$SU(2N)_D$	$SU(4)_Q$	$SU(4)_{\bar Q}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
a	$\Yasymm$	1	1	1	0	-4	0
$\bar a$	$\overline{\Yasymm}$	1	1	-1	0	-4	0
q	$\square$	$\bar\square$	1	0	1	$2N-2$	$\frac12$
$\bar q$	$\bar\square$	1	$\bar\square$	0	-1	$2N-2$	$\frac12$
M_k	1	$\square$	$\square$	0	0	$4N-4-8k$	1
H_m	1	$\Yasymm$	1	-1	2	$4N-8-8m$	1
$\bar H_m$	1	1	$\Yasymm$	1	-2	$4N-8-8m$	1

$$(30)$$

where $k = 0, \ldots, N-1$, $m = 0, \ldots, N-2$, and with the superpotential in the dual magnetic theory given by

$$W = \sum_k M_k q\bar q (a\bar a)^{N-1-k} + \sum_m \left(H_m q^2 a (a\bar a)^{N-2-m} + \bar H_m \bar q^2 \bar a (a\bar a)^{N-2-m} \right) \quad (31)$$

The 't Hooft anomaly matching conditions are again straightforward to check, while the mapping of flat directions is given by

$$
\begin{aligned}
Q\bar Q(A\bar A)^k &\leftrightarrow M_k & A^{N-1}Q^2 &\leftrightarrow a^{N-1}q^2 \\
Q^2\bar A(A\bar A)^k &\leftrightarrow H_k & \bar A^{N-1}\bar Q^2 &\leftrightarrow \bar a^{N-1}\bar q^2 \\
\bar Q^2 A(A\bar A)^k &\leftrightarrow \bar H_k & (A\bar A)^k &\leftrightarrow (a\bar a)^k \\
A^N &\leftrightarrow a^N & A^{N-2}Q^4 &\leftrightarrow a^{N-2}q^4 \\
\bar A^N &\leftrightarrow \bar a^N & \bar A^{N-2}\bar Q^4 &\leftrightarrow \bar a^{N-2}\bar q^4
\end{aligned}
$$

Note that a similarly constructed candidate for a self-dual to $SU(2N+1)$ with $\Yasymm + \overline{\Yasymm} + 4(\square + \bar\square)$ fails to match the global anomalies.

As a consistency check on the above duality for $SU(2N)$ we consider breaking $SU(2N)$ to $SU(2)^N$ by giving a VEV to the fields A and $\bar A$. The

result is N copies of the $SU(2)$ self-dual with 8 doublets presented in Section 2. Another check is to integrate out one flavor from the electric theory. On the magnetic side this corresponds to completely higgsing the gauge group since the operator $q(a\bar{a})^{N-1}\bar{q}$ is forced to have an expectation value by the M_0 equation of motion. The massless fields exactly correspond to the confining spectrum of $SU(2N)$ with $\Box\!\Box + \overline{\Box\!\Box} + 3(\Box + \overline{\Box})$ described in Ref. [11]. Again, part of the confining superpotential is reproduced by the tree-level superpotential of equation 31 while the rest is presumably generated by instanton effects.

While there is no self-dual for $SU(2N+1)$ with $\Box\!\Box + \overline{\Box\!\Box} + 4(\Box + \overline{\Box})$, one can derive a non-trivial dual for this theory by breaking the $SU(2N+2)$ gauge group of this self-dual to $SU(2N+1)$ with an expectation value for one flavor of quarks. The electric theory becomes $SU(2N+1)$ with $\Box\!\Box + \overline{\Box\!\Box} + 4(\Box + \overline{\Box})$, while the dual magnetic theory is still $SU(2N+2)$ with $\Box\!\Box + \overline{\Box\!\Box} + 4(\Box + \overline{\Box})$, but with a different combination of singlets and tree-level superpotential. Just like in the derived dualities of Section 3.2, the $SU(4) \times SU(4)$ global symmetry is not explicit in the magnetic theory, but only restored in the infrared.

3.5 $SO(N)$ with spinors and $N-4$ vectors

In this section, we present a series of self-dual theories, all of which have multiple self-duals. The theories we examine have $SO(N)$ gauge groups with $N-4$ vectors as well as some spinor representations. The duals of these theories contain the same gauge degrees of freedom as the electric theory, and some additional gauge singlets. The gauge singlet "meson" fields correspond to composite operators of the electric theory made up of two spinors and varying numbers of vectors. In the simplest self-dual, the $SO(N)$ vectors remain fundamentals of the $SU(N-4)$ global symmetry, while the dual spinors become antifundamentals under their global symmetry. Thus, in some sense only the spinors are being "dualized" in these duals.

As an explicit example we describe an $SO(8)$ theory with 4 spinors and 4 vectors in detail. This theory is particularly interesting because $SO(8)$ has a group automorphism. With this choice of matter content the automorphism implies a Z_2 symmetry which exchanges spinors and vectors. The duality is as follows:

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	$SO(8)$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
S	8_s	▢	1	1	$\frac{1}{4}$
V	8_v	1	▢	-1	$\frac{1}{4}$
	$SO(8)_{D1}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
s	8_s	$\overline{▢}$	1	1	$\frac{1}{4}$
v	8_v	1	▢	-1	$\frac{1}{4}$
M_0	1	▢▢	1	2	$\frac{1}{2}$
M_2	1	▤	▤	0	1
M_4	1	▢▢	1	-2	$\frac{3}{2}$

$$(32)$$

with a dual superpotential:

$$W_{\mathrm{magn}} = M_0 s^2 v^4 + M_2 s^2 v^2 + M_4 s^2. \tag{33}$$

Note that the "meson" content of this dual does not have a manifest Z_2 symmetry, thus the exchange symmetry is an accidental infrared symmetry in the dual description, in analogy to the symmetries of the $SU(2)$ theory discussed in Section 2. Again, there are a number of other self-duals, which are summarized in Table 2.

The operator mapping is:

$$
\begin{aligned}
S^2 &\leftrightarrow M_0 \\
V^2 &\leftrightarrow \bar{M}_0 \\
S^2 V^2 &\leftrightarrow M_2 \\
S^2 V^4 &\leftrightarrow M_4 \\
S^4 V^2 &\leftrightarrow \bar{M}_4
\end{aligned}
\tag{34}
$$

It is interesting to examine the possibility that this theory is in an interacting non-Abelian Coulomb phase, i.e. the gauge coupling runs to a non-trivial fixed point in the infrared. Then, due to the Z_2 symmetry, the anomalous dimensions of the spinor and vector quarks must be the same. This uniquely identifies the superconformal R symmetry (which is related to the scaling dimensions [4]) as the R symmetry whose charges we indicated in the tables above. Recalling the relation that the scaling dimension is $\frac{3}{2} R_{\mathrm{sc}}$ for fields in the superconformal algebra and the fact that gauge-invariant chiral superfields cannot have a dimension less than one (chiral superfields with R_{sc} less than $\frac{2}{3}$ are necessarily free-fields and decouple from the superconformal theory) we find that the chiral superfields M_0 and $\bar{M}_0$ are free in the infrared,

	$SO(8)_{D2}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
s	8_s	$\bar{\square}$	1	1	$\frac{1}{4}$
v	8_v	1	$\square$	-1	$\frac{1}{4}$
M_0	1	$\square\square$	1	2	$\frac{1}{2}$
M_4	1	$\square\square$	1	-2	$\frac{3}{2}$

	$SO(8)_{D3}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
s	8_s	$\square$	1	1	$\frac{1}{4}$
v	8_v	1	$\square$	-1	$\frac{1}{4}$
M_2	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	1

	$SO(8)_{D4}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
s	8_s	$\square$	1	1	$\frac{1}{4}$
v	8_v	1	$\bar{\square}$	-1	$\frac{1}{4}$
$\bar{M}_0$	1	1	$\square\square$	-2	$\frac{1}{2}$
M_2	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	1
$\bar{M}_4$	1	1	$\square\square$	2	$\frac{3}{2}$

	$SO(8)_{D5}$	$SU(4)$	$SU(4)$	$U(1)$	$U(1)_R$
s	8_s	$\bar{\square}$	1	1	$\frac{1}{4}$
v	8_v	1	$\bar{\square}$	-1	$\frac{1}{4}$
M_0	1	$\square\square$	1	2	$\frac{1}{2}$
$\bar{M}_0$	1	1	$\square\square$	-2	$\frac{1}{2}$
M_2	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	1
M_4	1	$\square\square$	1	-2	$\frac{3}{2}$
$\bar{M}_4$	1	1	$\square\square$	2	$\frac{3}{2}$

Table 2:   Multiple self-duals of $SO(8)$ with 4 spinors and 4 vectors.

group	content	"mesons"
$SO(4)$	$(8,8,0)$	$\sim SU(2) \times SU(2)$ with $8(\square,1) + 8(1,\square)$
$SO(5)$	$(8,1)$	$\sim Sp(4)$ with $8\,\square + \begin{array}{c}\square\\[-3pt]\square\end{array}$
$SO(6)$	$(4,4,2)$	$\sim SU(4)$ with $4(\square + \overline{\square}) + 2\begin{array}{c}\square\\[-3pt]\square\end{array}$
$SO(7)$	$(4,3)$	$s^2, s^2v, s^2v^2, s^2v^3$
$SO(8)$	$(4,0,4)$	s^2, s^2v^2, s^2v^4
$SO(8)$	$(2,2,4)$	$s^2, s^2v^2, s^2v^4, c^2, c^2v^2, c^2v^4, scv, scv^3$
$SO(8)$	$(3,1,4)$	$s^2, s^2v^2, s^2v^4, c^2, c^2v^4, scv, scv^3$
$SO(9)$	$(2,5)$	$s^2, s^2v, s^2v^2, s^2v^3, s^2v^4, s^2v^5$
$SO(10)$	$(2,0,6)$	s^2v, s^2v^3, s^2v^5
$SO(10)$	$(1,1,6)$	$s^2v, s^2v^5, c^2v, c^2v^5, sc, scv^2, scv^4, scv^6$
$SO(11)$	$(1,7)$	$s^2v, s^2v^2, s^2v^5, s^2v^6$
$SO(12)$	$(1,0,8)$	s^2v^2, s^2v^6

Table 3: The series of self-dual $SO(N)$ theories with $N-4$ vectors. The first column gives the gauge group, the second the matter content of the electric theory, while the third column gives the additional gauge singlet "mesons" one needs to include into the dual magnetic theory (with the appropriate tree-level superpotential). All of these theories turn out to have multiple dualities. As noted, the first three examples are equivalent to theories discussed in previous sections.

while the remaining degrees of freedom are interacting with scaling dimensions given by $\frac{3}{2}R_{\rm sc}$. Thus this theory may provide a realization of the exotic phenomena suggested in reference [12].

Similar self-duals can be found for numerous other $SO(N)$ theories with spinors and $N-4$ vectors. The properties of these duals are summarized in Table 3. We use the following notation for the matter content: (N_S, N_C, N_V), which denotes N_S spinors, N_C conjugate spinors, and N_V vectors. For $SO(N)$ groups with N odd only two numbers are given, the first for spinors, the second for vectors. The matter content of the self-dual theories is determined by assigning the vectors R-charge 0, and adding as many spinors as required for the spinors to have R-charge $\frac{1}{2}$.

Note that these self-duals flow to each other when giving a VEV to one vector. Both the electric and magnetic gauge groups break to $SO(N-1)$. One

vector is eaten, while $N-5$ vectors remain in the theory. It is straightforward to check, that the resulting theory is exactly the corresponding self-dual of $SO(N-1)$ in our table.

As a consistency check, one can give VEVs to all $SO(N)$ vectors, breaking $SO(N)$ to $SO(4) \sim SU(2) \times SU(2)$. It is easy to check that all self-duals of Table 3 reduce to two copies of one of the three $SU(2)$ self-duals discussed in Section 2.

Another check on these dualities consist of integrating out vectors from the theory. After integrating out one vector the electric theory confines without chiral symmetry breaking and with a confining superpotential (s-confines) [11]. The magnetic theory also confines, and it is straightforward to check that the confined degrees of freedom are correctly reproduced. However, just like in the duality of [9], the origin of some of the terms in the confining superpotential on the magnetic side is unknown, but may be due to instantons.

Finally, we consider giving masses to spinors in those theories where the spinors are in real representations. When integrating out all spinors the electric theory becomes $SO(N)$ with $N-4$ vectors, which has two branches of vacua [2]. One branch has a dynamically generated superpotential while the other branch is confining with no superpotential. To check how this arises in the magnetic theories we consider the example of $SO(6)$ with $(4, 4, 2)$ which is equivalent to $SU(4)$ with $2\,\square\!\square + 4(\square + \overline{\square})$. Integrating out the spinors in $SO(6)$ amounts to integrating out the $SU(4)$ flavors, which we do one at a time. Adding a mass term for one flavor gives non-zero VEVs to fields in the magnetic theory by the "meson" equation of motion which breaks the magnetic $SU(4)$ gauge group completely. After identifying the uneaten light degrees of freedom we obtain the confined spectrum and part of the confining superpotential of $SU(4)$ with $2\,\square\!\square + 3(\square + \overline{\square})$, described in Ref. [11]. The remaining part of the confining superpotential is presumably generated by instanton effects in the completely broken $SU(4)$ gauge group. Thus after integrating out one flavor the magnetic theory reproduces the confining theory with 3 flavors. It has been shown in Ref. [11], that integrating out the remaining three flavors does result in the correct description of the theory in both branches of vacua.

4　Conclusions

We have presented a large set of new self-dual $N = 1$ SUSY gauge theories. Starting from these self-duals one can obtain many new nontrivial dualities by integrating out flavors or by higgsing the gauge group. In addition to the derived dualities discussed here, a large class of duals for $SU(N)$ theories with antisymmetric tensors can be derived by giving expectation values to the spinor representations of our $SO(N)$ self-duals. A common feature of many of these derived dualities is that they have some accidental global symmetries, which emerge only in the infrared and are not explicit in the ultraviolet description. These accidental symmetries can make finding new dualities very difficult. Usually, one is looking for dual pairs by requiring that the global symmetries and their anomalies in the two theories match. When some of the global symmetries are accidental symmetries, present only in the infrared, we loose our most powerful tool for identifying dualities.

Another interesting feature of the presented dualities is that they do not seem to follow any obvious pattern. For example, $SU(5)$ with $2\,\square\!\square + 5(\square+\overline{\square})$ is dual to $SU(6)$ with $\square\!\square + 6(\square+\overline{\square})$, while $SU(5)$ with $2\,\square\!\square + 4(\square+\overline{\square})$ is dual to $Sp(4)$ with $\square\!\square + 10\,\square$. Just by changing the number of flavors, the gauge group and the matter content of the dual theory changes completely. Furthermore, theories with identical gauge groups and gauge degrees of freedom but with different "meson" content and superpotentials can be dual to very different theories. For example, $SU(6)$ with $\square\!\square + 6(\square + \overline{\square})$ is self-dual, dual to $SU(5)$ with $2\,\square\!\square + 5(\square + \overline{\square})$ or dual to $SU(4)$ with $2\,\square\!\square + 5(\square + \overline{\square})$, depending on the gauge singlet "meson" content and the tree-level superpotential.

These features of duality suggest that developing a systematic approach to finding $N = 1$ duals for theories with arbitrary matter content might be a very difficult task, which will likely require more insight into the dynamics of supersymmetric gauge theories than is presently available.

Acknowledgments

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